TECHNICAL APPENDICES

The real effects of monetary shocks in sticky price models: a sufficient statistic approach

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H Data Appendix

This appendix provides further empirical evidence. Section H.1 offers more summary statistics about the French CPI and a robustness-to-trimming exercise. Section H.2 explores the extent to which the statistical protocols used to measure prices are responsible for the small price changes, as suggested by Eichenbaum et al. (2014).

H.1 More statistics on the French CPI and robustness to trimming

Table 4 reports the frequency of price changes as well as selected moments of the distribution of price changes. The basic patterns that emerge form the CPI data (frequency of price change, average and standard deviation of price changes) match those documented by Berardi, Gautier, and Le Bihan (2015) for France and are representative of those obtaiend by Alvarez et al. (2006) for the Euro area. With the qualification that the frequency of price changes is typically found to be smaller in the Euro area than in the US, they also broadly match the US evidence by e.g. Nakamura and Steinsson (2008). The frequency of price change is around 17% per month, or about 2 price changes per year. The fraction of price decreases among price changes is around 40%. The average *absolute* price change (not reported in the table) is sizable (9.2%), as is the standard deviation of price changes (16.6%).

A second investigation on measurement error was developed by varying the upper and lower thresholds of small and large price changes used to define outliers. Results are displayed in Table 5. In each of the variants considered in Table 5, both kurtosis and the fraction of small price changes remain large. The lowest level of kurtosis obtains when we use the most stringent thresholds for outliers. If in the vein Eichenbaum et al. (2014) we consider that all price changes lower in absolute value that 1 percent are presumably measurement errors, and discard them from the sample, the resulting kurtosis is reduced to 7.1. Furthermore, some large price changes may as well be measurement (transcription) errors. If we restrict the sample to price changes larger in absolute value that 1 percent, and lower than log(2) (meaning that observaions with multiplied by more than 2, or divided by more than 2,– a stricter threshold than the 99^{th} percentile– are considered as likely measurement errors), the kurtosis is further reduced to 6.2.

H.2 Small price changes and measurement errors

This appendix examines to what extent the arguments of Eichenbaum et al. (2014) apply to our data and investigates the robustness of our findings to various criteria for trimming the data. Measurement errors may arise for several reasons. Eichenbaum, Jaimovich, and Rebelo

Table 4: Selected moments from the size distribution of price changes

	Dominick's DFF	CPI Data France all records	CPI Data France exc.sales
Frequency of price changes	32.7	17.1	15.7
Moments for the size of price changes: Δp_i			
Average	0.1	0.3	3.2
Standard deviation	24.5	15.6	11.9
Moments of standardized price changes: z			
Kurtosis	4.0	8.0	8.9
Number of obs. with $\Delta p_i \neq 0$	295,692	1,530,878	1,266,507

Source for Dominick's is Kilts Center for Marketing, data are weekly scanner price records for 400 weeks from 1989 to 1997. Source for CPI is INSEE, monthly price records from French CPI, data from 2003:4 to 2011:4. Coverage is around 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Frequency of price change is the average fraction of price changes per week (Dominick's) or month (INSEE) , in percent. Size of price change is the first-difference in the logarithm of price per unit, expressed in percent. Observations with imputed prices or quality change are discarded. Observations outside the interval $0.1 \leq |\Delta p_i| \leq P99$ are removed as outliers. "Exc. sales" exclude observations flagged as sales by the INSEE data collectors. Moments are computed aggregating all prices changes using CPI weights at the product level. The "kurtosis" row report kurtosis for the standardized price change $z_{ijt} = \frac{\Delta p_{ijt} - m_j}{\sigma_j}$ where m_j and σ_j are the mean and standard deviation of price changes in category j (see the text).

(2011) and Eichenbaum et al. (2014) articulate two concerns about the small price change. First they notice that in scanner data studies the price level of an item is typically computed as the ratio of recorded weekly revenues to quantity sold. To the extent that there are temporary or individual specific discounts (say coupons), this will generate spurious small price changes.[36] Moreover Eichenbaum et al. (2014) highlight a related problem for some CPI items: they spot 27 items (named ELIS in the BLS terminology) that are problematic because these prices are typically computed as a Unit Value Index (a ratio of expenditure to quantity purchased), or they are not consistently recorded in the same outlet, or they are the price of a bundle of goods (for instance the sum of airplane fare and airport tax). We were able to match these items with their counterparts in our French dataset. Out of the 27 problematic items 15 are not present in our data because in the French CPI those items are not recorded by a field agent but are centrally collected (thus not made available

[36] Notice that in principle CPI data are immune from this type of measurement error, as these data are direct transaction prices observed by a field agent. Indeed, in the instance of a temporary discount, the CPI dataset will record either no price change, or the large price change of observed during the discount, if the field agent happens to be collecting data during the temporary discount. Further, the protocol of data collection requires that the field agent records the price faced by a regular customer, not benefiting from individual-specific discounts.

Table 5: Robustness to trimming

Type of trimming	Freq	Kur(z)	Std(dp)	N obs	case
$\|\Delta p\| < P99$	17.05	7.98	15.63	1530878	0
exc sales	15.71	8.85	11.88	1266507	1
<log(10/3)	17.09	8.89	16.60	1542527	2
<log(2)	17.08	8.53	16.12	1537895	3
<100pct	16.86	7.03	13.33	1482044	4
>0	17.17	8.12	15.56	1540243	5
>0.5pct	16.63	7.61	15.86	1480753	6
>1pct	15.34	7.07	16.65	1377209	7
>1pct <log(2)	15.14	6.15	14.21	1328378	8
>1pct nosales	13.95	7.84	12.68	1116076	9

Source is INSEE, monthly price records from French CPI, data from 2003:4 to 2011:4. Coverage is aroud 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Freq. denotes monthly frequency of price change in percent. Size of price change Δp are the first-difference in the logarithm of price per unit, expressed in percent.Std(dp) is standard devation of log price change. Kur[z] denotes kurtosis of the distribution of standardized price changes. Standardized price changes are computed at the category of good * type of outlet level. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weigths at the product level. Each row describes a sub-sample constructed applying the filter described by the column "type of trimming". The subsample with flag "case 0" is the baseline sample in the main text of the paper: price changes are included if they are larger in absolute value than 0.1 percent, and lower in absolute value than the 99^{th} percentile; sales are included. Each subsequent row describes the impact of changing one (or two) of these thresholds and criteria, the one(s) that is explicitely mentionned. For example the second row considers the sample with $|\Delta p| > 0.1$, $|\Delta p| < P99$,and sales excluded; the third row considers the sample with $|\Delta p| > 0.1$, $|\Delta p| < log(10/3)$,and sales included "Ex. sales" exclude observations flagged as sales by the INSEE data collectors.

in the subset of CPI we have access to).[37] Concerning the 12 remaining items virtually no price record in the French CPI is computed as a Unit Value Index, which is hypothesized by Eichenbaum et al. (2014) as a major source of small price changes. Inspecting the patterns of price changes over these 12 potentially "problematic" items in our dataset shows that the amount of small price changes is not significantly different from the one detected over the rest of our sample. One exception is the price of "Residential water" where it can be suspected that many small variations in local taxes occur.[38]

Finally, Table 6 compares the fraction of small price changes in US vs the French data.

[37] These items are Hospital room in-patient; Hospital in-patient services other than room ; Electricity; Utility natural gas service; Telephone services, local charges ; Interstate telephone services ; Community antenna or cable TV ; Cigarettes; Garbage and trash collection; Airline fares; New cars; New trucks; Ship fares; Prescription drugs and medical supplies; Automobile insurance.

[38] Otherwise, on the bulk of consumption items, there are no local taxes in France, and the main, nation-wide, rate of the Value Added Tax rate did not move over the sample period.

Table 6: Fraction of small price changes: US and French CPI

Moments for the absolute value of price changes: $\lvert\Delta p_i\rvert$	France	US	Normal	Laplace
Average $\lvert\Delta p_i\rvert$	9.2	14.0		
Fraction of $\lvert\Delta p_i\rvert$ below 1%	11.8	12.5		
Fraction of $\lvert\Delta p_i\rvert$ below 2.5%	32.5	24.0		
Fraction of $\lvert\Delta p_i\rvert$ below 5%	57.1	40.6		
Fraction of $\lvert\Delta p_i\rvert$ below $(1/14) \cdot \mathbb{E}(\lvert\Delta p_i\rvert)$	2.4	12.5	4.5	6.9
Fraction of $\lvert\Delta p_i\rvert$ below $(2.5/14) \cdot \mathbb{E}(\lvert\Delta p_i\rvert)$	13.5	24.0	11.3	16.4
Fraction of $\lvert\Delta p_i\rvert$ below $(5/14) \cdot \mathbb{E}(\lvert\Delta p_i\rvert)$	28.7	40.6	22.4	30.0
Number of obs	1,542,586	1,047,547		

Note: For France the source is INSEE monthly price records from the French CPI (2003:4 to 2011:4). Coverage is around 65% of CPI weight since rents, and prices of fresh food and centrally collected items (e.g. electricity, train and airplane tickets) are not included in the dataset. Frequency of price change is monthly, in percent. Size of price change are the first-difference in the logarithm of price per unit, expressed in percent. Data are trimmed as in the baseline of Table 4. Observations with imputed prices or quality change are discarded. Moments are computed aggregating all prices changes using CPI weights at the product level. The US data are taken from Eichenbaum et al. (2014) Table 1, and refer to "Posted price changes" from 1998:1 to 2011:6. The mean absolute size of price changes is taken from Klenow and Kryvtsov (2008) table III where data are from 1998:1 to 2005:1. Figures for the US are weighted and cover around 70% of the CPI (US CPI includes owners equivalent rents, while French CPI does not). In the third panel we compute the threshold for defining small price changes as fraction of the mean so as to match the US figures in column 2 of the second panel. The Normal and Laplace distributions used in the last two columns have a zero mean and, without loss of generality, standard deviation equal to one.

The table uses the same thresholds of Eichenbaum et al. (2014) to measure the fraction of small price changes. The presence of small price changes (in absolute value) is at first sight a more prominent fact in France than in the US. One factor that may contribute to explaining this pattern is the fact that sales are less prevalent in France. Measurement error, as discussed above, may play a role. We nevertheless observe that, if we define small price changes as relative to the mean average price change, rather than with an absolute threshold, the fraction of small price change appears to be lower in France than in the US, as shown in Table 6.

I Details of the solution for the model with $n = 1$

Integrating the Bellman equation gives the following value function

$$V(p) = \frac{Bp^2 + \lambda V(0)}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + C\left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

where we already used that $V(p) = V(-p)$. Notice that the value function has a minimum (and zero derivative) at $p = 0$, which is the optimal return point. The constant C and the threshold value $\bar{p}$ are the values that solve the 2 equation system given by the value matching condition and the smooth pasting conditions.

The expected time to adjustment, $T(p)$ obeys the differential equation $\lambda T(p) = 1 + \frac{\sigma^2}{2} T''(p)$ with boundary condition $T(\bar{p}) = 0$. Given the symmetry of the law of motion for p, the function is symmetric, i.e. $T(p) = T(-p)$. Integrating gives $T(p) = \frac{1}{\lambda}\left(1 - \frac{e^{\sqrt{\frac{2\lambda}{\sigma^2}}p} + e^{-\sqrt{\frac{2\lambda}{\sigma^2}}p}}{e^{\sqrt{\frac{2\lambda}{\sigma^2}}\bar{p}} + e^{-\sqrt{\frac{2\lambda}{\sigma^2}}\bar{p}}}\right)$.

The distribution of price gaps $g(p)$ satisfies the Kolmogorov forward equation $0 = -\frac{2\lambda}{\sigma^2} g(p) + g''(p) \quad \text{for} \quad 0 < |p| \leq \bar{p}$. The density is symmetric, $g(p) = g(-p)$, and satisfies the boundary conditions: $g(\bar{p}) = 0$ and it integrates to one i.e. $2\int_0^{\bar{p}} g(p)\, dp = 1$ where we used that it is symmetric.[39]

J Proof that $\lim_{\bar{y}\to\infty} \xi(\sigma^2, r+\lambda, n, \bar{y}) = 0$

Note that, by examining the definition of κ_i and the sums in the expression for ξ we have that:

$$\lim_{\bar{y}\to\infty} \xi(\sigma^2, r+\lambda, n, \bar{y}) = \lim_{\bar{y}\to\infty} \xi\left(1, 1, n, \frac{(r+\lambda)\,\bar{y}}{\sigma^2}\right)$$

so this limit cannot depend on $r + \lambda$ or σ^2. Thus we denote it as:

$$\bar{\xi}(n) \equiv \lim_{\bar{y}\to\infty} \xi(1, 1, n, \bar{y})$$

So we have:

$$\bar{y} \approx \frac{\psi}{B}\,(r+\lambda)\,\left[1 - \bar{\xi}(n)\right] \quad \text{for large } \psi\,.$$

Now we show that $\bar{\xi}(n) = 0$. First we notice that the power series:

$$g(x) = \sum_{i=1}^{\infty} \prod_{s=1}^{i} \frac{1}{(s+2)(n+2s+2)}\; x^i$$

[39] The first boundary can be derived as the limit of the discrete time, discrete state, low of motion where each period is of length Δ and where p increases or decreases with probability 1/2, so that $g(p) = \frac{1}{2}g(p+\Delta) + \frac{1}{2}g(p-\Delta)$. At the boundary $\bar{p}$ this law of motion is $g(\bar{p}) = \frac{1}{2}g(\bar{p}-\Delta)$, which shows that $g(\bar{p}) \downarrow 0$ as $\Delta \downarrow 0$.

converges for all values of x since its coefficients satisfy the Cauchy-Hadamard inequality. Then we can write:

$$\xi(1,1,n,\bar{y}) \equiv \frac{\frac{2(n+2)}{\bar{y}}\frac{1}{g(\bar{y})} + \frac{1}{g(\bar{y})} + \frac{1}{\bar{y}^2}}{\frac{2(n+2)}{\bar{y}}\frac{1}{g(\bar{y})} + 2\frac{1}{g(\bar{y})} + \sum_{i=1}^{\infty} \omega(i,\bar{y})\ (2+i)}$$

where the weights $\omega(i,\bar{y})$ are given by:

$$\omega(i,x) = \frac{\frac{x^i}{\prod_{s=1}^{i}(s+2)(n+2s+2)}}{\sum_{j=1}^{\infty}\prod_{s=1}^{j}\frac{1}{(s+2)(n+2s+2)}\ x^j}$$

Note that for higher x the weights of smaller i decrease relative to the ones for higher i. Now since $g(\bar{y}) \to \infty$ as $\bar{y} \to \infty$, then:

$$\bar{\xi}(n) = \frac{1}{\lim_{\bar{y}\to\infty}\sum_{i=1}^{\infty}\omega(i,\bar{y})\ (2+i)}$$

To show that $\bar{\xi}(n) = 0$, suppose, by contradiction that is finite. Say, without loss of generality that equals $j+2$ for some integer j. Note that, by the form of the $\omega's$ and because $g(\bar{y})$ diverges as $\bar{y}$ gets large enough, then by any j and $\epsilon > 0$ there exist a y^* large enough so that $\sum_{i=1}^{j}\omega(i,\bar{y}) < \epsilon$ for any $\bar{y} > y^*$. Thus, the expected value must be larger than $2+j$.

Finally, we consider the case of $n \to \infty$. In this case we have that, the value function divided by n gives:

$$v = \min_T B \int_0^T \sigma^2\, t\, e^{-(\lambda+r)}\, dt + e^{-(r+\lambda)T}(\Psi + v)$$

where $\Psi = \lim_{n\to\infty}\psi/n$. The first order condition for T gives, for a finite T:

$$0 = \big(B\,\sigma^2\,T - (r+\lambda)\Psi\big) - (r+\lambda)e^{-(r+\lambda)T}v \tag{43}$$

Now consider the case where $\Psi \to \infty$. Note that v is finite since $T = \infty$, a feasible strategy as a finite value. Also let $\bar{Y} = \sigma^2 T = \lim_{n\to\infty}\frac{\bar{y}(n)}{n}$. Note that as $\Psi \to \infty$ then $\bar{Y}$ must also diverge towards ∞. Dividing the previous expression by Ψ:

$$\frac{\bar{Y}}{\Psi} = \frac{(r+\lambda)}{B} + (r+\lambda)e^{-(r+\lambda)T}\frac{v}{\Psi}$$

and taking the limits:

$$\lim_{\Psi\to\infty}\frac{\bar{Y}}{\Psi} = \frac{r+\lambda}{B} \quad . \quad \square$$

K Note on solutions of value function $v(y)$, expected time to adjust $\mathcal{T}(y)$ and invariant density of the squared price gap $f(y)$.

First we state a proposition which gives an explicit closed form solution to the value function $v(y)$ in the inaction region, i.e. for $y \in (0, \bar{y})$ subject to $v(0) < \infty$. The solution is parameterized by $\beta_0 = v(0)$.

Proposition 13 *Let $\sigma > 0$. The ODE in equation (5) is solved by the analytical function: $v(y) = \sum_{i=0}^{\infty} \beta_i \, y^i$, for $y \in [0, \bar{y}]$ where, for any β_0, the coefficients $\{\beta_i\}$ solve: $\beta_0 = \frac{n\sigma^2}{r}\beta_1$, $\beta_2 = \frac{(r+\lambda)\beta_1 - B}{2\sigma^2(n+2)}$, $\beta_{i+1} = \frac{r+\lambda}{(i+1)\sigma^2 \; (n+2i)} \; \beta_i$ for $i \geq 2$.*

The function described in this proposition allows to fully characterize the solution of the firm's problem. One can use it to evaluate the two boundary conditions described above, value matching and smooth pasting, and define a system of two equations in two unknowns, namely β_0 and $\bar{y}$.

The alert reader may have noticed that to solve for the invariant density f we have followed a standard procedure, i.e. set a 2nd order ordinary linear difference equation (the Kolmogorov forward equation) and find its solutions in terms of two constant, and using two boundary conditions to find the value of the constants. Instead to solve for v and $\mathcal{T}$ we have followed a different approach, we guess an infinite expansion around $y = 0$ and compute its coefficients. Additionally, it may have looked that we did not provide enough boundary conditions to be able to solve for $\mathcal{T}$ and v. For instance, for $\mathcal{T}$ we gave only one equation as boundary conditions, namely $\mathcal{T}(\bar{y}) = 0$. Here we explain that we could have followed the more standard route, which required an analysis of the behavior close to the $y = 0$ boundary, to set one constant to zero and also would have produced a less informative result, i.e. one in terms of modified Bessel functions. Nevertheless we include it here for completeness.

Note that $v(y)$, $\mathcal{T}(y)$ and $f(y)$ are solutions to a linear ODE on y whose homogeneous component, say $q(\cdot)$, solves :

$$y \, q''(y) + a \, q'(y) + b \, q(y) \; = \; 0 \tag{44}$$

for $y \in [0, \bar{y}]$, for (different) constants a and b, with different particular solution, and different boundary conditions. The general solution of the homogeneous equation (44) is given by:

$$q(y) = |b \, y|^{(1-a)/2} \left[C_1 I_\nu \left(2\sqrt{|b \, y|} \right) + C_2 K_\nu \left(2\sqrt{|b \, y|} \right) \right] \tag{45}$$

provided that $b\,y < 0$, i..e. that $b < 0$, where C_1 and C_2 are arbitrary constants, $\nu = |1-a|$ and where I_ν and K_v are the modified Bessel functions of the first and second kind respectively. The values of $b = -\lambda/(2\sigma^2)$ in the three cases. The value of $a = n/2$ for $\mathcal{T}$ and for v, which are the same Kolmogorov backward equation, and $a = -(n/2 - 2)$ for f, which is the Kolmogorov forward equation.

It is important to notice the behavior of $I_\nu(z)$ and $K_\nu(z)$ for values of $0 < z$ but very close to zero. We have:

$$I_\nu \backsim \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu \tag{46}$$

and

$$K_\nu \backsim \begin{cases} \frac{\Gamma(\nu+1)}{2}\left(\frac{2}{z}\right)^\nu & \text{if } \nu > 0 \\ -\log(z/2) - \gamma & \text{if } \nu = 0 \end{cases} \tag{47}$$

We thus have that each of the solution will behave as:

$$\begin{aligned} I_{|1-a|}\left(y^{1/2}\right)\ y^{(1-a)/2} &\backsim \frac{1}{\Gamma(|1-a|+1)}\left(\frac{y^{1/2}}{2}\right)^{|1-a|} y^{(1-a)/2} \\ &= \frac{1}{\Gamma(|1-a|+1)}\left(\frac{1}{2}\right)^{|1-a|} y^{(1-a)/2+|1-a|/2} \end{aligned}$$

So if $1-a = -|1-a|$, i.e. if $1-a \leq 0$, the value of this product is finite at $y \downarrow 0$. Otherwise it diverges to ∞. Likewise for $\nu = |1-a| > 0$:

$$\begin{aligned} K_{|1-a|}\left(y^{1/2}\right)\ y^{(1-a)/2} &\backsim \frac{\Gamma(|1-a|+1)}{2}\left(\frac{2}{y^{1/2}}\right)^{|1-a|} y^{(1-a)/2} \\ &= \frac{\Gamma(|1-a|+1)}{2}\left(\frac{2}{1}\right)^{|1-a|} y^{(1-a)/2-|1-a|/2} \end{aligned}$$

So if $1-a = |1-a|$, i.e. if $1-a \geq 0$, the value of this product is finite at $y \downarrow 0$. Otherwise it diverges to ∞. The case of $\nu = 0$ i.e. $a = 1$ is special, but $K_0(z)$ also diverges and $I_0(z)$ converges to a non-zero constant as $z \downarrow 0$.

Note that $v(0)$ and $\mathcal{T}(0)$ are both finite. For these two cases the Kolmogorov backward equation has $a = n/2$ so $1-a \geq 0$ iff $n \geq 2$. In these cases we have that C_2, the constant associated with K_ν must be zero. We can use the constant C_1 to impose the boundary condition $\mathcal{T}(\bar{y}) = 0$ for $\mathcal{T}$ and to have a one dimensional representation of v in the range of inaction given $\bar{y}$. Then we can use smooth pasting and value matching, i.e. two boundary conditions, to find the constants C_1 and $\bar{y}$.

Note that for f we don't require that $f(0)$ be zero, since the density at zero gap can be infinite if the y mean reverts to zero fast enough. Thus in this case we will, in general, have both constants be non-zero.

L Power series representation of Kurtosis

Given $(\lambda, \sigma^2, \bar{y})$ the kurtosis of the steady state price distribution can be written as:

$$Kur\left(\Delta p_i\right) = \frac{Q(0)}{\frac{\sigma^4}{N(\Delta p_i)^2}} = \frac{\left(\lambda/\sigma^2\right)^2 Q(0)}{\left(\mathcal{L}(\phi, n)\right)^2}$$

where $Q(y)$ is the expected fourth moment at the time of adjustment τ conditional on having today a squared price gap y, i.e.

$$Q(y) = \mathbb{E}\left(\Delta p_i^4(\tau)\,|\,y(0) = y\right) = \frac{3}{(n+2)n}\;\mathbb{E}\left(y^2(\tau)|y(0) = y\right)$$

where $y(\tau)$ is the value of the squared price gap at the stopping time and where, using results from Alvarez and Lippi (2014), we have that $Kur(\Delta p_i|y) = \frac{3n}{(n+2)}$ and the variance is $Var\left(\Delta p_i\,|\,||p||^2 = y\right) = y/n$. Notice that for $y \in [0, \bar{y}]$ the function $Q(y)$ obeys the o.d.e.:

$$\lambda Q(y) = \lambda \frac{3y^2}{(n+2)n} + Q'(y)\,n\sigma^2 + Q''(y)\,2\sigma^2 y$$

with boundary condition $Q(\bar{y}) = \frac{3\bar{y}^2}{(n+2)n}$. Assuming that $Q(y) = \sum_{i=0}^{\infty} a_i y^i$, matching coefficients, and writing them as function of a_0 one obtains:

$$\begin{aligned} a_1(a_0) &= \frac{a_0}{\frac{\sigma^2}{\lambda}n}\;,\; a_2(a_0) = \frac{a_1(a_0)}{2\frac{\sigma^2}{\lambda}(n+2)}\;,\; a_3(a_0) = \frac{a_2(a_0) - \frac{3}{(n+2)n}}{3\frac{\sigma^2}{\lambda}(n+4)} \text{ and} \\ a_{i+1}(a_0) &= \frac{a_i(a_0)}{(i+1)\frac{\sigma^2}{\lambda}(n+2i)} \text{ for } i \geq 3 \end{aligned}$$

Thus $Q(0) = a_0$ is determined as the solution to $\sum_{i=0}^{\infty} a_i(a_0)\,\bar{y}^i = \frac{3\,\bar{y}^2}{(n+2)n}$. After tedious but simple algebra this gives:

$$Q(0) = a_0 = \frac{3n}{(n+2)}\left(\frac{\sigma^2}{\lambda}\right)^2 \left[\frac{\phi^2 + \sum_{i=3}^{\infty}\left(\prod_{j=3}^{i}\frac{n}{j[n+2(j-1)]}\right)\phi^i}{1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{n}{j[n+2(j-1)]}\right)\phi^i}\right]$$

where $\phi \equiv \frac{\lambda \bar{y}}{n\sigma^2}$.

Replacing $Q(0)$ into $Kur\left(\Delta p_i\right) = \frac{\left(\lambda/\sigma^2\right)^2 Q(0)}{\mathcal{L}^2}$ and using equation (6) for $\mathcal{L}(\phi, n)$ we get

$$Kur\left(\Delta p_i\right) = \frac{3n}{(n+2)} \left[\frac{\phi^2 + \sum_{i=3}^{\infty}\left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i}{1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i}\right] \left(\frac{1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i}{\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i}\right)^2$$

Thus

$$Kur\left(\Delta p_i\right) = \frac{3n}{(n+2)} \frac{\left(\phi^2 + \sum_{i=3}^{\infty}\left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i\right)\left(1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i\right)}{\left(\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i\right)^2} \tag{48}$$

For future reference note that $Kur\left(\Delta p_i\right)/N\left(\Delta p_i\right) = (1/\lambda)\mathcal{L}(\phi, n)\, Kur\left(\Delta p_i\right)$ so

$$\frac{Kur\left(\Delta p_i\right)}{N\left(\Delta p_i\right)} = \frac{1}{\lambda}\frac{3n}{(n+2)} \frac{\left(\phi^2 + \sum_{i=3}^{\infty}\left(\prod_{j=3}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i\right)}{\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i} \frac{n}{j[n+2(j-1)]}\right)\phi^i}$$

Using that

$$\prod_{j=1}^{i} \frac{n}{j\left[n+2(j-1)\right]} = \frac{(n/2)^i}{i!}\prod_{j=1}^{i}\frac{1}{\left[\frac{n}{2}+(j-1)\right]} = \frac{(n/2)^i}{i!}\frac{\Gamma\left(\frac{n}{2}\right)}{\Gamma\left(\frac{n}{2}+i\right)}$$

we can write:

$$\begin{aligned}
\frac{Kur\left(\Delta p_i\right)}{N\left(\Delta p_i\right)} &= \frac{1}{\lambda}\frac{3n}{(n+2)} \frac{\left(\phi^2 + 2\,\Gamma\left(\frac{n}{2}+2\right)\left(\frac{2}{n}\right)^2 \sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\Gamma\left(\frac{n}{2}\right)\,\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \frac{1}{\lambda}\frac{3n}{(n+2)} \frac{2\,\Gamma\left(\frac{n}{2}+2\right)\left(\frac{2}{n}\right)^2}{\Gamma\left(\frac{n}{2}\right)} \frac{\left((1/2)\,\left(1/\Gamma\left(\frac{n}{2}+2\right)\right)\left(\frac{n}{2}\right)^2\phi^2 + \sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \frac{1}{\lambda}\frac{12n}{(n+2)} \frac{2\,\left(n/2+1\right)\left(n/2\right)}{n^2} \frac{\left(\frac{1}{2\Gamma\left(\frac{n}{2}+2\right)}\left(\phi\, n/2\right)^2 + \sum_{i=3}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i\right)}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i} \\
&= \frac{6}{\lambda}\frac{\sum_{i=2}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i}{\sum_{i=1}^{\infty}\frac{1}{i!\,\Gamma\left(\frac{n}{2}+i\right)}\left(\phi\, n/2\right)^i}
\end{aligned}$$

which gives the right hand side of equation (19). From there it is apparent that for a fixed

n, this ratio is increasing in $\lambda \bar{y}/\sigma^2$ and that for a fixed $\lambda \bar{y}/\sigma^2$, this ratio is increasing in n.

M Proof of Lemma 4

Proof. (of Lemma 4.) We use the property of the n independent BM's to write m as a function of a pair (z, y), where $z = \sum_i p_i$, as well as to write g as a function of (z, y) only. If each price gap follows an independent BM with common variance per unit of time σ^2, then, applying Ito's Lemma one can show that the pair (y, z) follows:

$$\begin{aligned}
\mathrm{d}y(t) &= n\sigma^2 \mathrm{d}t + 2\sigma\sqrt{y(t)}\, \mathrm{d}\mathcal{W}^a(t) \\
\mathrm{d}z(t) &= \sqrt{n}\sigma \left[\frac{z(t)}{\sqrt{n\, y(t)}}\, \mathrm{d}\mathcal{W}^a(t) + \sqrt{1 - \left(\frac{z(t)}{\sqrt{n\, y(t)}}\right)^2}\, \mathrm{d}\mathcal{W}^b(t) \right]
\end{aligned}$$

where $\mathcal{W}^a, \mathcal{W}^b$ are 2 standard (univariate) independent BM's. So that $\mathbb{E}(\mathrm{d}y)^2 = 4\sigma^2 y\, \mathrm{d}t$, $\mathbb{E}(\mathrm{d}z)^2 = n\sigma^2\, \mathrm{d}t$, and $\mathbb{E}(\mathrm{d}z\, \mathrm{d}y) = 2\sigma^2 z\, \mathrm{d}t$.

Hence we can write $\tilde{m}(p_1, ..., p_n) = \tilde{m}\left(||p||^2, \sum_{i=1}^n p_i\right)$, in which case $\tilde{m}$ solves the PDE :

$$\lambda \tilde{m}(z, y) = -z + \tilde{m}_y(z, y) n\sigma^2 + \tilde{m}_{zz}(z, y)\frac{n\sigma^2}{2} + \tilde{m}_{yy}(z, y)\frac{4\sigma^2 y}{2} + \tilde{m}_{zy}(z, y)\, 2\sigma^2 z$$

with boundary conditions: $\tilde{m}(z, \bar{y}) = 0$. We guess, and verify, that $\tilde{m}(z, y) = z\, \kappa_n(z)$ for some function $\kappa_n(\cdot)$ and where for emphasis we include the subindex n indicating the number of products. We then obtain:

$$\lambda \kappa_n(y) = -1 + \kappa_n'(y)\,(n+2)\sigma^2 + \kappa_n''(y)\, 2\, \sigma^2\, y$$

for all $0 \leq y \leq \bar{y}$ and $\kappa_n(\bar{y}) = 0$. Note that, except of the sign, this function obeys the same ODE and boundary conditions than the one for the time until adjustment $\mathcal{T}_{n+2}(y)$, which we solved to obtain $\mathcal{L}$ as if there were $n+2$ products instead of n products, and hence we get:

$$\kappa_n(y) = -\mathcal{T}_{n+2}(y) \tag{49}$$

The joint density of the invariant distribution $h(z, y)$ can be written as:

$$h(z, y) = s(z|y)\, f(y)$$

where f is the invariant distribution of y and $s(z|y)$ is the density distribution of the sum of the coordinates of a uniform distribution on an n dimensional hypersphere with square norm equal to y. In Alvarez and Lippi (2014) we have shown that this distribution is given by

$$s(z|y) = \frac{1}{Beta\left(\frac{n-1}{2},\frac{1}{2}\right)\ \sqrt{n\,y}} \left(1-\left(\frac{z}{\sqrt{y\,n}}\right)^2\right)^{(n-3)/2} \quad \text{for } \ z \in (-\sqrt{y\,n}\,,\ \sqrt{y\,n}) \tag{50}$$

Thus we can write

$$\mathcal{M}(\delta) \ = \ \frac{1}{\epsilon}\int_0^{\bar{y}}\int_{-\sqrt{ny}}^{\sqrt{ny}} \frac{1}{n}\,\tilde{m}(z-n\delta, y-2z\delta+n\delta^2)\ h(z,y)\ dz\ dy \tag{51}$$

and where we can express the invariant distribution of (z, y) with density h. Differentiating this expression w.r.t. δ and evaluating it at $\delta = 0$:

$$\begin{aligned}\mathcal{M}'(0) \ &= \ -\frac{1}{n\epsilon}\int_0^{\bar{y}}\int_{-\sqrt{ny}}^{\sqrt{ny}}\left[n\,\frac{\partial \tilde{m}(z,y)}{\partial z} \ + \ \frac{\partial \tilde{m}(z,y)}{\partial y}\,2\,z\right]\ h(z,y)\ dz\ dy \\ &= \ -\frac{1}{n\epsilon}\left[\int_0^{\bar{y}} n\,\kappa_n(y)\,f(y)\,dy + 2\int_0^{\bar{y}}\kappa_n'(y)\int_{-\sqrt{ny}}^{\sqrt{ny}}\ z^2\ s(z|y)dz\,f(y)\ dy\right].\end{aligned}$$

Integrating z^2 w.r.t. s gives $\int_{-\sqrt{ny}}^{\sqrt{ny}} z^2\ s(z|y)dz = y$ so

$$\begin{aligned}\mathcal{M}'(0) \ &= \ -\frac{1}{n\epsilon}\int_0^{\bar{y}}[\ n\,\kappa_n(y) + 2\,\kappa_n'(y)y\ ]\,f(y)\ dy \\ &= \ \frac{1}{\epsilon}\int_0^{\bar{y}}\left[\mathcal{T}_{n+2}(y) + \frac{2}{n}\,\mathcal{T}_{n+2}'(y)\,y\ \right]\ f(y)\ dy\end{aligned}$$

where the last equality uses equation (49). □

N Power series representation of $\mathcal{T}_{n+2} + \mathcal{T}'_{n+2}\,y\,(2/n)$

Lemma 4 shows that $\partial m/\partial\delta$ can be written in terms of $\mathcal{T}_{n+2}$, the expected time until a price adjustment, as characterized in Proposition 3. In that proof we obtain the power series representation

$$\mathcal{T}_{n+2}(y) = \sum_{i=0}^{\infty}\alpha_{i,\,n+2}\,y^i$$

with

$$\alpha_{1,n+2} = \frac{1}{(\sigma^2/\lambda)(n+2)}\alpha_{0,n+2} - \frac{1}{\sigma^2(n+2)} = \frac{1}{(\sigma^2/\lambda)(n+2)}\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right]$$

and for $i \geq 1$:

$$\alpha_{i+1,\,n+2} = \frac{\alpha_{i,\,n+2}}{(i+1)\,(\sigma^2/\lambda)\,(n+2+2i)} = \frac{\alpha_{i,\,n+2}}{(i+1)\,(\sigma^2/\lambda)\,(n/2+1+i)}\,\frac{1}{2}\,\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right].$$

and using the properties of the Γ function:

$$\alpha_{i,\,n+2} = \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda}{2\sigma^2}\right)^i\left[\alpha_{0,n+2} - \frac{1}{\lambda}\right]$$

Note that $\mathcal{T}_{n+2}(0) = \alpha_{0,\,n+2}$

Given the power series representation we have for all $y \in [0, \bar{y}]$:

$$\mathcal{T}_{n+2}(y) + \mathcal{T}'_{n+2}(y)\,y\,\frac{2}{n} = \sum_{i=0}^{\infty}\alpha_{i,\,n+2}\left[1 + i\frac{2}{n}\right]\,y^i$$

$$= \quad \alpha_{0,n+2} + \left[\alpha_{0,n+2} - \frac{1}{\lambda}\right]\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left[1 + i\frac{2}{n}\right]\left(\frac{\lambda y}{2\sigma^2}\right)^i$$

Note that $\alpha_{0,n+2} = \mathcal{T}_{n+2}(0)$ with

$$\lambda\,\alpha_{0,n+2} = \ell = \frac{\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1}{j[n+2+2(j-1)]}\right)\left(\frac{\lambda\bar{y}}{\sigma^2}\right)^i}{1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1}{j[n+2+2(j-1)]}\right)\left(\frac{\lambda\bar{y}}{\sigma^2}\right)^i} = \frac{\sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1}{j\left[\frac{n}{2}+j\right]}\right)\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}{1 + \sum_{i=1}^{\infty}\left(\prod_{j=1}^{i}\frac{1}{j\left[\frac{n}{2}+j\right]}\right)\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}$$

$$= \frac{\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}$$

Thus we have:

$$\lambda\left(\mathcal{T}_{n+2}(y)+\mathcal{T}'_{n+2}(y)\,y\,\frac{2}{n}\right)=\frac{\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}$$

$$+\left[\frac{\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i}-1\right]\left[\sum_{i=1}^{\infty}\left[1+i\frac{2}{n}\right]\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda y}{2\sigma^2}\right)^i\right]$$

$$=\frac{\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{1}{2}\right)^i\left(\frac{\lambda\bar{y}}{\sigma^2}\right)^i-\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left[1+i\frac{2}{n}\right]\left(\frac{\lambda y}{2\,\sigma^2}\right)^i}{\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{1}{2}\right)^i\left(\frac{\lambda\bar{y}}{\sigma^2}\right)^i}$$

$$=\frac{\sum_{i=1}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left[\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i-\left(1+\frac{2i}{n}\right)\left(\frac{\lambda y}{2\sigma^2}\right)^i\right]}{\sum_{i=0}^{\infty}\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+i+1\right)}\left(\frac{\lambda\bar{y}}{2\,\sigma^2}\right)^i} \tag{52}$$

We can write this as:

$$\lambda\left(\mathcal{T}_{n+2}(y)+\mathcal{T}'_{n+2}(y)\,y\,\frac{2}{n}\right)=\frac{\sum_{i=1}^{\infty}\gamma_i}{\sum_{i=0}^{\infty}\gamma_i}-\frac{\sum_{i=1}^{\infty}\gamma_i\left(1+\frac{2\,i}{n}\right)\left(\frac{y}{\bar{y}}\right)^i}{\sum_{i=0}^{\infty}\gamma_i} \tag{53}$$

where

$$\gamma_i=\frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)}\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i \tag{54}$$

O Power series representation of the density $f(y)$

From equation (7) we can write f as the product of a power of y and the sums of two modified Bessel functions of the first and second kind, multiplied by appropriate constants.

Consider then $n\geq 3$ and n odd, so that $\nu=n/2-1$ is not an integer. When n is even the expression for K_ν requires to evaluate the limit, so it is more complicated. Thus, we can write:

$$\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)}I_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right)\;=\;\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)}\sum_{i=0}^{\infty}\beta_{i,\frac{n}{2}-1}\left(\frac{\lambda y}{2\sigma^2}\right)^i$$

where

$$\beta_{i,\frac{n}{2}-1}\equiv\frac{1}{i!\,\Gamma(i+n/2)}$$

and for ν not an integer

$$\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)} K_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right) = \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i - \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{4}-\frac{1}{2}\right)} I_{\frac{n}{2}-1}\left(2\sqrt{\frac{\lambda y}{2\sigma^2}}\right)$$

where

$$\beta_{i,1-\frac{n}{2}} \equiv \frac{1}{i!\,\Gamma(i+2-n/2)}$$

This means we can write:

$$f(y) = \left(C_I - \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} C_K\right) \left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i + C_K \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i$$

Since $f(0) > 0$ and

$$f(0) = C_K \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \beta_{0,1-\frac{n}{2}} = C_K \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} \frac{1}{\Gamma(2-n/2)}$$

then $C_K > 0$. Then to set $f(\bar{y}) = 0$ we obtain:

$$\frac{C_I - \frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} C_K}{-\frac{\frac{\pi}{2}}{\sin\left((\frac{n}{2}-1)\pi\right)} C_K} = \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}$$

Using the expressions for $f(0)$ and $f(\bar{y}) = 0$ we can then rewrite f as:

$$f(y) = -f(0)\,\Gamma\left(2-\frac{n}{2}\right) \left(\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i\right) \times \left[\frac{\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}\right]$$

Using that $1 = \int_0^{\bar{y}} f(y)dy$ we obtain an expression for $f(0)$ and replacing in the previous formula we obtain:

$$f(y) = \left[\frac{\left(\frac{\lambda y}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda y}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} \right] \Bigg/ \left[\frac{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \frac{\bar{y}}{i+n/2} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \frac{\bar{y}}{i+1} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i} \right] \tag{55}$$

Remark. While this expression was obtained for $n \geq 2$ and n odd, it does work for any real number $n \geq 2$ different from an even natural. Since it is continuous in n, the expression equation (55) can be used to obtain the values of f in the case of n is even by taking the limit as n approaches any even natural, or by evaluating at a real number very close to the desired even natural number.

P Discrete Time Formulation for Proposition 12.

We start with discrete time version of the process for price gaps, with length of the time period Δ, which makes some of the arguments more accessible. Let N be

$$N(t+\Delta) = \begin{cases} N(t) & \text{with probability } (1-\lambda\Delta) \\ N(t)+1 & \text{with probability } \lambda\Delta \end{cases} \tag{56}$$

Thus, as $\Delta \downarrow 0$ this process converges to a continuous time Poisson counter with instantaneous intensity rate λ per unit of time. Let $\bar{p}_i$ follow n drift-less random walks

$$\bar{p}_i(t+\Delta, p) = \begin{cases} \bar{p}_i(t,p) \; + \; \sigma\sqrt{\Delta} & \text{with probability } 1/2 \\ \bar{p}_i(t,p) \; - \; \sigma\sqrt{\Delta} & \text{with probability } 1/2 \end{cases} \tag{57}$$

where the initial condition satisfies:

$$\bar{p}_i(0) = p_i \text{ for } i = 1, .., n \; ,$$

and where the n random walks are independent of each other and of the Poisson counter. As $\Delta \downarrow 0$ the process for $\bar{p}$ converges to a Brownian motion whose changes have variance σ^2 per unit of time. We define the stopping time of the first price adjustment $\tau(p)$, conditional on the starting at price gap vector p at time zero, as:

$$\begin{aligned}
\tau_1 &\equiv \min\left\{t = 0, \Delta, 2\Delta, \ldots : N(j\Delta + \Delta) - N(j\Delta) = 1\right\}, \\
\tau_2(p) &\equiv \min\left\{t = 0, \Delta, 2\Delta, \ldots : \sum_{i=1}^{n} \left(\bar{p}_i(j\Delta + \Delta, p)\right)^2 \geq \bar{y}\right\} \text{ and} \\
\tau(p) &\equiv \min\left\{\tau_1 \,,\, \tau_2(p)\right\}.
\end{aligned}$$

The function g is the density for the continuous time limit, i.e. the case where $\Delta \downarrow 0$. For small Δ, we can approximate the distribution of the *fraction* of firms with price gap vector p as the product of the density g and a correction to convert it into a probability, i.e a fraction. This gives:

$$g\left(p_1, \ldots, p_; n, \lambda/\sigma^2, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n$$

where the last term uses that in each dimension price gaps vary discretely in steps of size $\sigma\sqrt{\Delta}$. We can write the discrete time impulse response function as:

$$\mathcal{P}(t, \delta; \sigma, \lambda, \bar{y}, \Delta) \;=\; \Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) + \sum_{s=\Delta}^{t} \theta(\delta, s; \sigma, \lambda, \bar{y}, , \Delta)\, \Delta\,,$$

In this expression we can, without loss of generality, restrict t to be an integer multiple of Δ. We have divided the expression for θ by Δ, and hence multiplied its contribution back by Δ in $\mathcal{P}$, so that it has the interpretation of the contribution per unit of time to the IRF of price changes at time t, i.e. it has the units of a density. Moreover, in this manner the term has a non-zero limit, and the expression in $\mathcal{P}$ converges to an integral. Thus we get the $\mathcal{P} = \lim \mathcal{P}(\Delta)$ as $\Delta \downarrow \infty$. The functions θ and Θ are given by:

$$\Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) \equiv \sum_{||p(0)-\iota\delta|| \geq \bar{y}} \left(\delta - \frac{\sum_{j=0}^{n} p_j(0)}{n}\right) g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n \text{ , and}$$

$$\theta(\delta, t; \sigma, \lambda, \bar{y}, \Delta) \equiv$$
$$-\frac{1}{\Delta} \sum_{||p(0)-\iota\delta|| < \bar{y}} \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j(t, p)}{n} \mathbf{1}_{\{\tau(p)=t\}} \;\middle|\; p = p(0) - \iota\delta\right] g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n$$

Time scaling of the IRF with $N(\Delta p_i)$. For this (i) Note that if multiply the parameters σ^2 and λ by a constant $k > 0$, leaving $\bar{y}$ unaltered, then $N(\Delta p_i)' = k\, N(\Delta p_i)$, where primes are used to denote the values that correspond to the scaled parameters. This follows directly from the expression we derive for $N(\Delta p_i) = 1/T(0)$ in Proposition 3. (ii) By Proposition 4 with these changes the distribution of price changes implied by $(\sigma^2, \lambda, \bar{y})$ is exactly the same as the one implied by $(k\sigma^2, k\lambda, \bar{y})$. (iii) we change notation and write $(\sigma^2, \lambda, \bar{y})$ instead of $(\lambda, \sigma^2, \psi/B)$ and omit n. We establish that

$$\mathcal{P}_n\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}\right) = \mathcal{P}_n\left(t, \delta; \sigma^2, \lambda, \bar{y}\right)$$

We will do so by establishing this proposition for the discrete time version of the IRF. Yet the result is immediate, since λ and σ^2 are the only two parameters which are rates per unit of time (the other parameters are n and $\bar{y}$), so by multiplying them by k we just scale time. The details can be found in the discrete time formulation, whose notation we develop below. We show that

$$\mathcal{P}(t, \delta; k\sigma^2, k\lambda, \bar{y}, \Delta/k) = \mathcal{P}(t/k, \delta; \sigma^2, \lambda, \bar{y}, \Delta) \tag{58}$$

We will do so by establishing this proposition for the discrete time version of the IRF. Let $\Delta' = \Delta/k$, $\sigma'^2 = \sigma^2 k$ and $\lambda' = \lambda k$. Note that, by construction $\sigma' \sqrt{\Delta'} = \sigma \sqrt{\Delta}$ and $\lambda'/(\sigma')^2 = \lambda/(\sigma)^2$. To establish this we first note that, for a given shock δ, Θ depends only on n, $\bar{y}$, $\sigma\sqrt{\Delta}$, and λ/σ^2. This is because the invariant density g and the scaling factor to convert it into probabilities depends only on those parameters. Second we show that

$$\sum_{s=\Delta/k}^{t/k} \frac{\Delta}{k} \theta\left(s, \delta; k\sigma^2, k\lambda, \bar{y}, \frac{\Delta}{k}\right) = \sum_{s=\Delta}^{t} \Delta\, \theta\left(s, \delta; \sigma, \lambda, \bar{y}, \Delta\right)$$

This follows because for each s and $p(0)$

$$\begin{aligned} & \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\left(s, p\right)}{n} \mathbf{1}_{\{\tau(p)=s\}} \;\middle|\; p = p(0) - \iota\delta\,;\, \sigma, \lambda, \Delta\right] \\ = \;& \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\left(\frac{s}{k}, p\right)}{n} \mathbf{1}_{\left\{\tau(p)=\frac{s}{k}\right\}} \;\middle|\; p = p(0) - \iota\delta\,;\, \sigma', \lambda', \Delta'\right] \end{aligned}$$

where we include the parameters $(\lambda, \sigma^2, \Delta)$ as argument of the expected values. This itself follows because, using equation (56) and equation (57) then the processes for $\{\bar{p}_i\}$ are the same in the original time and in the time time scales by k since the probabilities of the counter to go up $\lambda'\Delta' = \lambda\Delta$ and the steps of the symmetric random walks $\sigma'\sqrt{\Delta'} = \sigma\sqrt{\Delta}$ are the same in the original time and the time scaled by k. In particular we have that

$$\bar{p}_j\left(\frac{s}{k}, p; \lambda', \sigma'^2, \Delta'\right) \equiv \bar{p}_j\left(\frac{s}{k}, p; k\lambda, k\sigma^2, \frac{\Delta}{k}\right) = \bar{p}_j\left(s, p; \lambda, \sigma^2, \Delta\right) = \hat{p}$$

with exactly the same probabilities for each price gap $\hat{p} \in \mathbb{R}$ and each time $s \geq 0$. Also, repeating the arguments used for Θ, we have $g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n = g\left(p(0); n, \frac{\lambda'}{\sigma'^2}, \bar{y}\right) \left(\sigma'\sqrt{\Delta'}\right)^n$. Thus, since equation (58) holds for all $\Delta > 0$, taking limits

$$\mathcal{P}\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}\right) = \lim_{\Delta \downarrow 0} \mathcal{P}\left(\frac{t}{k}, \delta; k\sigma^2, k\lambda, \bar{y}, \frac{\Delta}{k}\right) = \lim_{\Delta \downarrow 0} \mathcal{P}\left(t, \delta; \sigma^2, \lambda, \bar{y}, \Delta\right) = \mathcal{P}\left(t, \delta; \sigma^2, \lambda, \bar{y}\right)$$

Scaling of the IRF in the monetary shock with $Std(\Delta p_i)$. For this we use properties of the invariant distribution f, which are then inherited by g. In particular, we will compare the IRF with parameters $(\lambda, \sigma^2, \bar{y})$ with one with parameters $(\lambda', \sigma'^2, \bar{y})$ where $\lambda' = \lambda$, $\sigma'^2 = k\,\sigma^2$ and $\bar{y}' = k\,\bar{y}$. With this choice we have $N(\Delta p_i)' = N(\Delta p_i)$ and thus $\ell = \lambda'/N(\Delta p_i)'$ since $\lambda\bar{y}/(n\sigma^2) = \lambda'\bar{y}'/(n\sigma'^2)$ (see Proposition 3). Then by Proposition 1 we have that the standard deviation of price changes scales up with k, i.e.: $Std(\Delta p_i)' = \sqrt{k}\, Std(\Delta p_i)$. The main idea is that the invariant distribution corresponding to the $'$ parameters is a radial expansion of the original, so that $\int_0^y f(x; \lambda, \sigma^2, \bar{y})dx = \int_0^{yk} f(x; \lambda', \sigma'^2, \bar{y}')dx$ and thus $f(y, \lambda, \sigma^2, \bar{y}) = k f(yk, \lambda', \sigma'^2, \bar{y}')$. Indeed using Lemma 3 we have:

$$f\left(y; \frac{\lambda}{\sigma^2}, \bar{y}\right) = k\,f\left(yk; \frac{\lambda}{k\sigma^2}, k\bar{y}\right) \equiv k\,f\left(yk; \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \quad . \tag{59}$$

Thus we have:

$$\begin{aligned} g\left(p_1, ..., p_; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) &= f\left(p_1^2 + \cdots + p_n^2; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \frac{\Gamma\left(n/2\right)}{2\,\pi^{n/2}\ \left(p_1^2 + \cdots + p_n^2\right)^{(n-2)/2}} = \\ &= kf\left(k(p_1^2 + \cdots + p_n^2); n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \frac{\Gamma\left(n/2\right) k^{(n-1)/2}}{2\,\pi^{n/2}\ \left(k(p_1^2 + \cdots + p_n^2)\right)^{(n-2)/2}} \\ &= g\left(\sqrt{k}(p_1, ..., p_n); n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) k^{(n-2)/2}\, k \end{aligned}$$

Using this for the discrete time formulation we have:

$$\begin{aligned} g\left(p; n, \frac{\lambda}{\sigma^2}, \bar{y}\right) \left(\sigma\sqrt{\Delta}\right)^n &= g\left(\sqrt{k}\,p; n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \left(\sigma'\sqrt{\Delta}\right)^n k^{(n-2)/2}\,k\,k^{-n/2} \\ &= g\left(\sqrt{k}\,p; n, \frac{\lambda'}{\sigma'^2}, \bar{y}'\right) \left(\sigma'\sqrt{\Delta}\right)^n \end{aligned}$$

Note that $\{||p(0) - \iota\delta|| \geq \bar{y}\} = \{||\sqrt{k}\,p(0) - \iota\sqrt{k}\,\delta|| \geq \sqrt{k}\,\bar{y}\} = \{||\sqrt{k}\,p(0) - \iota\delta'|| \geq \bar{y}'\}$. Also

$$\left(\delta - \frac{\sum_{j=0}^n p_j(0)}{n}\right)\sqrt{k} = \left(\delta' - \frac{\sum_{j=0}^n \sqrt{k}\,p_j(0)}{n}\right)$$

Thus

$$\sqrt{k} \sum_{||p(0)-\iota\delta|| \geq \bar{y}} \left(\delta - \frac{\sum_{j=0}^{n} p_j(0)}{n} \right) g\left(p(0); n, \frac{\lambda}{\sigma^2}, \bar{y} \right) \left(\sigma\sqrt{\Delta} \right)^n$$

$$= \sum_{||\sqrt{k}p(0)-\iota\delta'|| \geq \bar{y}'} \left(\delta' - \frac{\sum_{j=0}^{n} \sqrt{k} p_j(0)}{n} \right) g\left(\sqrt{k}\, p(0); n, \frac{\lambda'}{\sigma'^2}, \bar{y}' \right) \left(\sigma'\sqrt{\Delta} \right)^n$$

Using the definition of $\Theta(\cdot, \Delta)$:

$$\sqrt{k}\, \Theta(\delta; \sigma, \lambda, \bar{y}, \Delta) = \Theta\left(\sqrt{k}\, \delta; k\sigma^2, \lambda, k\bar{y}, \Delta \right) \equiv \Theta\left(\delta'; \sigma'^2, \lambda', \bar{y}'\Delta \right) .$$

Since this holds for all Δ, by taking limits as $\Delta \downarrow 0$, we have shown the desired result for Θ. The result for θ follows the steps for g. We set $\Delta' = \Delta$ and note that for all $p(0) \in \mathbb{R}^n$, scaling factor $k > 0$ and time horizon $s > 0$:

$$\sqrt{k}\; \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\,(s, p)}{n}\; \mathbf{1}_{\{\tau(p)=s\}} \;\middle|\; p = p(0) - \iota\delta\,;\, \sigma, \lambda, \Delta \right]$$

$$= \mathbb{E}\left[\frac{\sum_{j=0}^{n} \bar{p}_j\,(s, p)}{n}\; \mathbf{1}_{\{\tau(p)=s\}} \;\middle|\; p = \sqrt{k}\, p(0) - \iota\delta'\,;\, \sigma', \lambda', \Delta \right] .$$

This follows because $\lambda' = \lambda$ and $\sigma'\sqrt{\Delta'} = \sqrt{k}\,\sigma\,\sqrt{\Delta}$, thus the each $p \in \mathbb{R}^n$ the paths $\sqrt{k}\,\bar{p}(s, p; \sigma, \lambda) = \bar{p}(s, \sqrt{k}p; \sigma', \lambda')$ occur with the same probabilities.

Q Detailed Proof. of Proposition 8.

Proof. (of Proposition 8.) In general we have $\underline{\delta} = 2\sqrt{\bar{y}/n}$, since for a shock of this size every single firm for which $||p||^2 = y \leq \bar{y}$ before the shock will find that $||p - \iota\delta||^2 \geq \bar{y}$, where ι is a vector of ones. In particular we want to find out the smallest value of δ for which

$$||p - \iota\delta||^2 = ||p||^2 - 2\,\delta \sum_i p_i + n\delta^2 \geq \bar{y}$$

for any $||p||^2 \leq \bar{y}$. Using that $\sum_i p_i \leq n\sqrt{y/n}$ for $y = ||p||^2$ it is easy to establish the desired result.

We can rewrite it as $\underline{\delta} = 2\sqrt{\bar{y}/n} = 2\sqrt{\sigma^2/\lambda}\,\sqrt{\phi}$, which gives an equivalent way to write the expression for $\underline{\delta}$ as

$$\underline{\delta} = Std(\Delta p_i)\, 2\sqrt{\frac{\phi}{\mathcal{L}(\phi,n)}} \text{ where } \phi \equiv \bar{y}\,\lambda/(n\sigma^2)\ .$$

where $\phi(n,\ell) \equiv \bar{y}\lambda/(n\sigma^2)$ a function that depends only on ℓ and n, as shown in Proposition 3. Using Proposition 1 we have:

$$(N(\Delta p_i)/\lambda)\ Var(\Delta p_i) = \sigma^2/\lambda \text{ or } \sigma^2/\lambda = Var(\Delta p_i)/\ell$$

Combining the two equations we obtain the desired result.

Note that $\phi(\ell,n)/\ell = \phi/\mathcal{L}(\phi,n)$. Since $\mathcal{L}(\phi,n)$ is increasing in ϕ with $\lim_{\phi\to\infty}\mathcal{L}(\phi,n) = 1$, then $\lim_{\ell\to 1}\phi(\ell,n)/\ell = \infty$. To study the limit as $\ell \to 0$, using the functional form of $\mathcal{L}$, and taking a Taylor expansion of $\mathcal{L}(\phi,n) = \phi + o(\phi)$, thus

$$\frac{\phi}{\mathcal{L}(\phi,n)} = \frac{\phi}{\phi + o(\phi)} = \frac{1}{1 + o(\phi)/\phi}\,,$$

and hence

$$\lim_{\ell\to 0}\frac{\phi(\ell,n)}{\ell} = \lim_{\phi\to 0}\frac{\phi}{\mathcal{L}(\phi,n)} = 1\,.$$

Omitting n to simplify the notation we have:

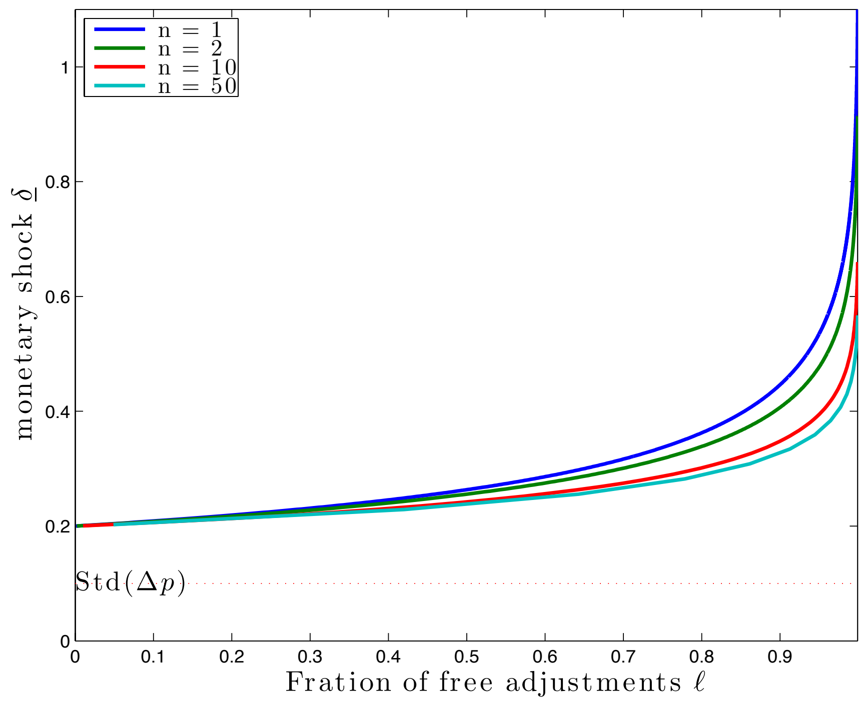
$$\frac{\partial}{\partial\phi}\left[\frac{\phi}{\mathcal{L}(\phi)}\right] = \frac{1}{\mathcal{L}(\phi)}\left[1 - \frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)}\right]$$

and rewriting $\mathcal{L}(\phi) = \frac{g(\phi)}{1+g(\phi)}$ we obtain: $\mathcal{L}'(\phi) = \frac{g'(\phi)}{[1+g(\phi)]^2}$ and thus

$$\frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)} = \frac{g'(\phi)}{(1+g(\phi))^2}\frac{(1+g(\phi))}{g(\phi)}\phi = \frac{g'(\phi)}{(1+g(\phi))}\frac{\phi}{g(\phi)}$$

since $g(\cdot)$ is convex and $g(0) = 0$ then $0 = g(0) \geq g(\phi) + g'(\phi)(0-\phi)$ or $g(\phi) \leq g'(\phi)\phi$ $\frac{\mathcal{L}'(\phi)\phi}{\mathcal{L}(\phi)} \leq \frac{1}{1+g(\phi)} \leq 1$ and thus $\phi(\ell,n)/\ell$ is strictly increasing in ℓ for all $\ell \in (0,1)$. $\square$

Figure 7: Minimum size of monetary shock for full price flexibility



R Proof of Lemma 5

Proof. (of Lemma 5) We can rewrite this expression as

$$\frac{\lambda\, Kur\,(\Delta p_i)}{6\, N\,(\Delta p_i)} = \frac{\sum_{i=1}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i \frac{1}{1+i}}{\sum_{i=0}^{\infty} \frac{\Gamma\left(\frac{n}{2}+1\right)}{i!\,\Gamma\left(\frac{n}{2}+1+i\right)} \left(\frac{\lambda \bar{y}}{2\sigma^2}\right)^i \frac{1}{1+i}} = \frac{\sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}}{\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}} \tag{60}$$

Thus the equation

$$\frac{\lambda\, Kur\,(\Delta p_i)}{6\, N\,(\Delta p_i)} = \int_0^{\bar{y}} \left[\lambda \left(\mathcal{T}_{n+2}(y) + \mathcal{T}'_{n+2}(y)\, y\, \frac{2}{n}\right)\right] f(y)\, dy \tag{61}$$

is equivalent to:

$$\frac{\sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}}{\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}} - \frac{\sum_{i=1}^{\infty} \gamma_i}{\sum_{i=0}^{\infty} \gamma_i} = -\frac{\sum_{i=1}^{\infty} \gamma_i \left(1 + \frac{2\,i}{n}\right)}{\sum_{i=0}^{\infty} \gamma_i} \int_0^{\bar{y}} \left(\frac{y}{\bar{y}}\right)^i f(y)\, dy$$

We can write this equation as:

$$\frac{\left(\sum_{i=0}^{\infty} \gamma_i\right)\left(\sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty} \gamma_i\right)\left(\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}\right)}{\left(\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}\right)\left(\sum_{i=0}^{\infty} \gamma_i\right)} = -\frac{\sum_{i=1}^{\infty} \gamma_i \left(1+\frac{2\,i}{n}\right)}{\sum_{i=0}^{\infty} \gamma_i} \int_0^{\bar{y}} \left(\frac{y}{\bar{y}}\right)^i f(y)\,dy$$

or

$$\frac{\left(\gamma_0 + \sum_{i=1}^{\infty} \gamma_i\right)\left(\sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty} \gamma_i\right)\left(\gamma_0 + \sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}\right)}{\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}} = -\left[\sum_{i=1}^{\infty} \gamma_i \left(1+\frac{2\,i}{n}\right)\right] \int_0^{\bar{y}} \left(\frac{y}{\bar{y}}\right)^i f(y)\,dy$$

or

$$\frac{\gamma_0\left(\sum_{i=1}^{\infty} \gamma_i \frac{1}{1+i}\right) - \left(\sum_{i=1}^{\infty} \gamma_i\right)\gamma_0}{\sum_{i=0}^{\infty} \gamma_i \frac{1}{1+i}} = -\left[\sum_{i=1}^{\infty} \gamma_i \left(1+\frac{2\,i}{n}\right)\right] \int_0^{\bar{y}} \left(\frac{y}{\bar{y}}\right)^i f(y)\,dy$$

and using that $\gamma_0 = 1$ and rearranging:

$$\sum_{i=1}^{\infty} \frac{\gamma_i \frac{1}{1+i}}{\sum_{j=0}^{\infty} \gamma_j \frac{1}{1+j}}\, i = \left[\sum_{i=1}^{\infty} \gamma_i \left(1+\frac{2\,i}{n}\right)\right] \int_0^{\bar{y}} \left(\frac{y}{\bar{y}}\right)^i f(y)\,dy \tag{62}$$

Using the expression for f, and solving the integrals of terms by term we have:

$$\sum_{j=1}^{\infty} \frac{\gamma_j \frac{1}{1+j}}{\sum_{s=0}^{\infty} \gamma_s \frac{1}{1+s}}\, j = \sum_{j=1}^{\infty} \gamma_j \left(1+\frac{2\,j}{n}\right) \times \tag{63}$$
$$\left(\left[\frac{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \frac{\bar{y}}{\frac{n}{2}+i+j} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \frac{\bar{y}}{i+1+j} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}\right] \Big/ \right.$$
$$\left.\left[\frac{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \frac{\bar{y}}{\frac{n}{2}+i} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}-1\right)} \sum_{i=0}^{\infty} \beta_{i,\frac{n}{2}-1} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i} - \frac{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \frac{\bar{y}}{i+1} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}{\sum_{i=0}^{\infty} \beta_{i,1-\frac{n}{2}} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i}\right]\right)$$

canceling the values of $\bar{y}$, and defining

$$\xi_i = \frac{1}{i!\,\Gamma\left(i+\frac{n}{2}\right)} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^{\left(\frac{n}{2}+i-1\right)} \quad \text{and} \quad \rho_i = \frac{1}{i!\,\Gamma\left(i+2-\frac{n}{2}\right)} \left(\frac{\lambda\bar{y}}{2\sigma^2}\right)^i$$

$$\sum_{j=1}^{\infty} \frac{\gamma_j \frac{1}{1+j}}{\sum_{s=0}^{\infty} \gamma_s \frac{1}{1+s}} j = \sum_{j=1}^{\infty} \gamma_j \left(1 + \frac{2\,j}{n}\right) \times \tag{64}$$

$$\left(\left[\frac{\sum_{i=0}^{\infty} \xi_i \frac{1}{\frac{n}{2}+i+j}}{\sum_{i=0}^{\infty} \xi_i} - \frac{\sum_{i=0}^{\infty} \rho_i \frac{1}{i+1+j}}{\sum_{i=0}^{\infty} \rho_i} \right] \Big/ \left[\frac{\sum_{i=0}^{\infty} \xi_i \frac{1}{\frac{n}{2}+i}}{\sum_{i=0}^{\infty} \xi_i} - \frac{\sum_{i=0}^{\infty} \rho_i \frac{1}{i+1}}{\sum_{i=0}^{\infty} \rho_i} \right] \right)$$

S Detailed Proof. of Proposition 7.

First we turn to the steady state firm's problem considered in Section 3.2. In that firm's problem we use the same discount rate r for any inflation rate μ. The reason for this is that the period return function is itself normalized by nominal wages which we assume that growth at a constant rate μ and that the nominal rate is equal to $r + \mu$, so that these two effect cancel. The price gap p_i is a real quantity, the difference between the ideal markup and the current markup, and has drift equal to minus the inflation rate due to the increase in the nominal wages. The period return is still $B||p||^2 \equiv B\,y$, but each of the product's price gap evolve as $\mathrm{d}p_i(t) = -\mu\,\mathrm{d}t + \sigma\,\mathrm{d}W_i(t)$. In this problem it is not longer true that y is sufficient to index the state of the firm's problem, since the distribution of $y(t+dt)$ cannot be computed only knowing $y(t)$. While in Alvarez and Lippi (2014) we show that one can take the state to be (y, z) where z is the sum of the price gaps: $z = \sum_{i=1}^{n} p_i$, for the arguments here we keep the entire price gap vector $p \in \mathbb{R}^n$ as the state. In this case the inaction set is no longer a hyper-sphere, nor is the optimal return point to set a zero price gap for each of the products. We let $\mathcal{I}(\mu) \subset \mathbb{R}^n$ be the inaction set –so the firm adjust only if it receives a free adjustment opportunity or if it exist the inaction set. We regard $\mathcal{I}(z)$ as a correspondence parametrized by μ, and let $\hat{p}(\mu) \in \mathbb{R}^n$ be the optimal return point -which is identical across all products- a function parametrized by μ. Note that for any rectangle $\subset\mathbb{R}^n$ the uncontrolled price gaps satisfy that $\Pr\{p(t) - p(0) \in \mathfrak{p} \,|\, \mu\} = \Pr\{-(p(t) - p(0)) \in \mathfrak{p} \,|\, -\mu\}$. This equality uses that the increments of a standard brownian motion are normally distributed. Using this property, and the symmetry around zero of the period return function, it is easy to show that $\hat{p}(\mu) = -\hat{p}(\mu)$. Also, one can see that if $p \in \mathcal{I}(\mu)$ then it must be the case that $-p \in \mathcal{I}(-\mu)$. From these two properties of the decision rules one concludes that $N(\Delta p_i)(\mu)$ and that any even centered moment of the distribution of the price changes, and hence its ratio such as kurtosis $Kur(\Delta p_i; \mu)$, is symmetric around $\mu = 0$. The same property is shown in Alvarez, Lippi, and Paciello (2011) for a closely related model. Likewise, the (negative) symmetry of $\mathcal{M})(\delta, \mu)$ follows by considering first the invariant distribution of price gaps, and then the dynamics of each one. For the invariant distribution of price gaps as defined in Section D, whose density is denoted by $g(p; \mu)$, we note that $g(p; \mu) = g(-p; -\mu)$ –where we now indexed the density only by the inflation rate μ, allowing the optimal decision rule to change with it. Following the same steps we can construct the impulse response of prices $\mathcal{P}(t, \delta; \mu)$ which we index in the same way as the density. We define this impulse response as the change in price level t periods after a once and for all shock δ to the path of the level of money that has occurred to an economy starting at the steady state distribution of price gaps. The price level is in $\mathcal{P}(\delta, t; \mu)$ is measured relative to what the prices would have been absence of a shock, where they would have been rising at a constant rate μ. Using the results previously established we have: $\mathcal{P}(t, -\delta; -\mu) = -\mathcal{P}(t, \delta; \mu)$. Using this property of the impulse response of the price level into definition of $\mathcal{M}$ in equation (12), we obtain the desired (negative) symmetry of this function.

Second, we sketch the differences in the GE set-up when $\mu \neq 0$. In this case the same arguments yields that both nominal interest rates and wages growth at a constant rate μ independently of the distribution of prices at time zero. Additionally, the nominal profit function of the firm, once we replace the first order condition for the households for consumption, labor, and money, can be written as a function of the price gap (i.e. the deviation relative to the markup that maximizes static profits) and the period nominal wages. Hence, one can approximate the real profits (deflated by the money supply) in the same way as with zero inflation, obtaining the same second order approximation. Finally, the result in Proposition 7 in Alvarez and Lippi (2014) which states that GE feedback effects are of order higher than second order in the firm's problem applies almost with no changes.

T Algebraic details for the Proof of Proposition 9

To compute the probabilities $P(t|i)$ notice that

$$P_1\left(t+dt \,|\, i\right) = \left(1-\theta_1 dt\right)\, P_1\left(t \,|\, i\right) + \theta_0\, dt\, \left[1 - P_1\left(t \,|\, i\right)\right]$$

for $i \in \{0, 1\}$. Taking a Taylor expansion in $P_1\left(t+dt \,|\, 1\right)$, dividing by dt, canceling terms we get:

$$P_1'\left(t \,|\, i\right) = -\left(\theta_1 + \theta_0\right)\, P_1\left(t \,|\, i\right) + \theta_0$$

The solution of this o.d.e. is:

$$P_1\left(t \,|\, i\right) = \frac{\theta_0}{\theta_0 + \theta_1} + B\, e^{-(\theta_0+\theta_1)t}$$

for some constant B. Evaluating this solution at $t = 0$ we have:

$$P_1\left(0 \,|\, i\right) = \frac{\theta_0}{\theta_0 + \theta_1} + B \text{ or } B = P_1\left(0 \,|\, i\right) - \frac{\theta_0}{\theta_0 + \theta_1}$$

Thus the solution is

$$P_1(t\,|\,i) = \frac{\theta_0}{\theta_0+\theta_1} + \left[P_1(0\,|\,i) - \frac{\theta_0}{\theta_0+\theta_1}\right] e^{-(\theta_0+\theta_1)t}$$

By definition we have

$$P_1(0\,|\,0) = 0 \text{ and } P_1(0\,|\,1) = 1$$

which gives the desired expressions.

Next , notice that:

$$\mathbb{E}_0\left[\sigma(t)^2|u(0)=i\right] = \sigma_0^2\,[1-P_1(t|i)] + \sigma_1^2\,P_1(t|i)$$

and

$$\mathbb{E}_0\left[\sigma(t)^2\right] = \mathbb{E}_0\left[\sigma(t)^2|u(0)=1\right]\frac{\theta_0}{\theta_0+\theta_1} + \mathbb{E}_0\left[\sigma(t)^2|u(0)=0\right]\frac{\theta_1}{\theta_0+\theta_1} \quad .$$

Grouping all terms:

$$\mathbb{E}_0\left[\sigma(t)^2\right] = \sigma_0^2 + \left(\sigma_1^2-\sigma_0^2\right)\left[P_1(t\,|\,0)\frac{\theta_1}{\theta_1+\theta_0} + P_1(t\,|\,1)\frac{\theta_0}{\theta_1+\theta_0}\right] \quad .$$

We can now use the expressions for $P_1(t\,|i)$ to obtain:

$$\mathbb{E}_0\left[\sigma(t)^2\right] = \sigma_0^2 + \\ \left(\sigma_1^2-\sigma_0^2\right)\left[\frac{\theta_0}{\theta_0+\theta_1}\left[1-e^{-(\theta_0+\theta_1)t}\right]\frac{\theta_1}{\theta_1+\theta_0} + \frac{\theta_0}{\theta_0+\theta_1}\left[1+\frac{\theta_1}{\theta_0}e^{-(\theta_0+\theta_1)t}\right]\frac{\theta_0}{\theta_1+\theta_0}\right] \quad .$$

Note that we can take common factor $e^{-(\theta_0+\theta_1)t}$ in the right hand side and obtain equation (31).

We also have, using the law of iterated expectations, for $0 \le s \le t \le T$:

$$k(t,s) = \mathbb{E}_0\left[\,\sigma(t)^2\,\sigma(s)^2\,\right] = \mathbb{E}_0\left[\,\sigma(s)^2\,\mathbb{E}\left[\sigma(t)^2\,|\,\sigma^2(s)\right]\right]$$

which we can write as:

$$\mathbb{E}_0\left[\,\sigma(t)^2\,\sigma(s)^2\,\right] = \mathbb{E}_0\left[\,\bar{\sigma}_i^2\,\mathbb{E}\left[\sigma(t)^2\,|\,u(s)=i\right]\right]$$

for which we have the inner expectation:

$$\mathbb{E}\left[\sigma(t)^2\,|\,u(s)=i\right] = \sigma_0^2 + \left(\sigma_1^2-\sigma_0^2\right)P_1(t-s\,|\,i)$$

and we also have

$$\mathbb{E}_0\left[\,\sigma(t)^2\,\sigma(s)^2\,\right]=\mathbb{E}\left[\,\sigma(t)^2\,\sigma(s)^2\,|u(0)=1\right]\frac{\theta_0}{\theta_0+\theta_1}+\mathbb{E}\left[\,\sigma(t)^2\,\sigma(s)^2\,|u(0)=0\right]\frac{\theta_1}{\theta_0+\theta_1}\quad .$$

Thus we can write (after some algebra):

$$\begin{aligned}&\mathbb{E}\left[\,\sigma(t)^2\,\sigma(s)^2\,|u(0)=i\right]\\&=\sigma_0^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,0\right)\right]\left(1-P_1\left(s\,|\,i\right)\right)+\sigma_1^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,1\right)\right]\,P_1\left(s\,|\,i\right)\quad .\end{aligned}$$

Finally, taking expected values for the initial $u(0)$, and using the formula above, we get:

$$\begin{aligned}&k(t,s)\\&=\sigma_0^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,0\right)\right]\left[1-P_1\left(s\,|\,1\right)\frac{\theta_0}{\theta_0+\theta_1}-P_1\left(s\,|\,0\right)\frac{\theta_1}{\theta_0+\theta_1}\right]\\&+\sigma_1^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,1\right)\right]\,\left[P_1\left(s\,|\,0\right)\frac{\theta_1}{\theta_0+\theta_1}+P_1\left(s\,|\,1\right)\frac{\theta_0}{\theta_1+\theta_1}\right]\end{aligned}$$

and using the expressions for the probabilities, or the definition of ergodicity,

$$P_1\left(s\,|\,0\right)\frac{\theta_1}{\theta_0+\theta_1}+P_1\left(s\,|\,1\right)\frac{\theta_0}{\theta_1+\theta_1}=\frac{\theta_0}{\theta_1+\theta_0}$$

we get:

$$k(t,s)=\sigma_0^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,0\right)\right]\frac{\theta_1}{\theta_1+\theta_0}+\sigma_1^2\,\left[\sigma_0^2+\left(\sigma_1^2-\sigma_0^2\right)P_1\left(t-s\,|\,1\right)\right]\,\frac{\theta_0}{\theta_1+\theta_0}\quad .$$

Replacing the expression for $P_1\left(t-s\,|\,1\right)$ and rearranging we get equation (32).

U A model with two point non-zero random fixed cost

This version of the model assumes that with probability λ per unit of time the menu cost is smaller than the regular adjustment, namely that it costs $b\,\psi$ with $b\in(0,1)$. For simplicity we focus here on a model with one product, i.e. $n=1$. The introduction of a small (but non zero) adjustment cost will imply that this model will have a lower bound for the size of price changes $\underline{p}>0$, such that no $|\Delta p_i|<\underline{p}$ will be observed. in spite of this important change, which may be important to fit cross section data, we stress that the formula in equation (1) continues to hold.

Firm's problem. The firm's optimal policy now involves two thresholds: $0 < \underline{p} < \bar{p}$. If the price gap is small, i.e. if $|p| \in [0, \underline{p}]$ the firm optimally decides not to adjust the price, even if an opportunity for cheap adjustment occurs. If the price gap is large, i.e. if $|p| \in [\underline{p}, \bar{p})$, the firm adjusts the price only if a cheap adjustment opportunity arises. As in the case where $b = 0$, the firm adjust its price the first time that $|p|$ reaches $\bar{p}$.

Given the values of two thresholds $\underline{p}, \bar{p}$, the value function v can be describe as two functions holding in each segment, as follows:

$$
\begin{aligned}
r\, v_0(p) &= B\, p^2 + \frac{\sigma^2}{2}\, v_0''(p), \quad \text{for } p \in [0, \underline{p}]\ , \\
r\, v_1(p) &= B\, p^2 + \lambda\left[v_0(0) + b\psi - v_1(p)\right] + \frac{\sigma^2}{2}\, v_1''(p), \quad \text{for } p \in [\underline{p}, \bar{p}]
\end{aligned}
$$

where we use that the optimal return point upon adjustment is $v_0(0)$ and where used that by symmetry $v_i(p) = v_i(-p)$ for $i = 0, 1$.

The value function can be expressed as the sum of a particular solution and two solutions multiplied by constants K_0 and K_1 and the two parameters $0 < \underline{p}, \bar{p}$. The value function has the following boundary conditions $v_0(\underline{p}) = v_1(\underline{p})$ and $v_0(0) + \psi = v_1(\bar{p})$, as well as the smooth pasting conditions $v_0'(\underline{p}) = v_1'(\underline{p})$ and $0 = v_1'(\bar{p})$. Using the four boundary conditions one solve for both the value function (i.e. the constants K_i) and the thresholds $\underline{p}, \bar{p}$. We give the details in Appendix U.1 and Appendix U.2.

Frequency of price changes. To find the frequency of price changes we first introduce the expected time to adjustment function $T(p)$. This function obeys the following ODE:

$$
0 = 1 + \frac{\sigma^2}{2} T_0''(p) \quad \text{for} \ \ 0 < |p| \leq \underline{p} \quad \text{and} \quad \lambda T_1(p) = 1 + \frac{\sigma^2}{2} T_1''(p) \quad \text{for } \underline{p} < |p| \leq \bar{p}
$$

with $T_i(p) = T_i(-p)$, and boundary conditions $T_0(\underline{p}) = T_1(\underline{p})$, $T_0'(\underline{p}) = T_1'(\underline{p})$ and $T_1(\bar{p}) = 0$. Thus

$$
T_0(p) = J - \frac{p^2}{\sigma^2} \ \ \text{and} \ \ T_1(p) = \frac{1}{\lambda} + K e^{\varphi |p|} + L e^{-\varphi |p|}
$$

where the J, K, L are constant to be determined using the boundary conditions, and where $\varphi = \sqrt{2\,\lambda / \sigma^2}$. Thus, given thresholds $\underline{p}, \bar{p}$, solving for the function T boils down to solve three linear equations in three unknowns as detailed in Appendix U.4. In particular the average number of adjustment per period is simply:

$$
N\left(\Delta p_i\right) = \frac{1}{T_0(0)} = \frac{1}{J}\,, \tag{65}
$$

Kurtosis of price changes. To measure the steady state kurtosis of price changes, we first solve for the density function for the price gaps $g(p) \in [0, \bar{p}]$. This density solves

$$
\begin{aligned}
0 &= g_0''(p) \text{ for } 0 \leq |p| \leq \underline{p} \quad \text{and} \quad 0 = -\frac{2\lambda}{\sigma^2} g_1(p) + g_1''(p) \text{ for } \underline{p} < |p| \leq \bar{p} \qquad \text{or} \\
g_0(p) &= C_1 + C_2 \, |p| \text{ for } 0 \leq |p| \leq \underline{p} \quad \text{and} \quad g_1(p) = C_3 e^{\varphi|p|} + C_4 e^{-\varphi|p|} \text{ for } \underline{p} \leq |p| \leq \bar{p}
\end{aligned}
$$

where the 4 constants solve the 4 equations $g_0(\underline{p}) = g_1(\underline{p})$, $g_0'(\underline{p}) = g_1'(\underline{p})$, $g_1(\bar{p}) = 0$ and $1/2 = \int_0^{\underline{p}} g_0(p)\, \mathrm{d}p + \int_{\underline{p}}^{\bar{p}} g_1(p)\, \mathrm{d}p$ which use that the density is differentiable. Given $\underline{p}, \bar{p}$ the solution boils down to solve four linear equations in four unknowns, as detailed in Appendix U.5.

Then using that only the fraction $2\int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p$ of cheap adjustment opportunities will trigger an actual price change, the distribution of (non-zero) price changes $p \in [-\bar{p}, -\underline{p}] \cup [\underline{p}, \bar{p}]$ is symmetric and is given by (we only report the formulas for $x > 0$) . Thus the distribution of (positive) price changes is

$$
\text{Price changes} \sim \begin{cases} \text{density for a price change of size } p \in [\underline{p}, \bar{p}) & : \frac{\lambda}{N_a} g_1(p) \\ \text{mass point at } \bar{p} & : \frac{1}{2} - \frac{\lambda}{N_a} \int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p \end{cases}
$$

The $j - th$ moment of price changes for j even is

$$
\mathbb{E}\left(\Delta p^j\right) = \frac{\lambda}{N_a} 2 \int_{\underline{p}}^{\bar{p}} x^j \, g_1(p) \, \mathrm{d}p + \left(1 - \frac{\lambda \, 2 \int_{\underline{p}}^{\bar{p}} g_1(p)\mathrm{d}p}{N_a}\right) \bar{p}^j
$$

Using that $Var\left(\Delta p\right) N\left(\Delta p\right) = \sigma^2$, the kurtosis of price changes is given by:

$$
Kur\left(\Delta p\right) = \frac{\mathbb{E}\left(\Delta p^4\right)}{\left(\sigma^2 / N\left(\Delta p\right)\right)^2} \,. \tag{66}
$$

Area under impulse response. To find an expression for $\mathcal{M}'(0)$ we first define the contribution to the area under impulse response of a firm that starts with price gap p. Letting $m(p)$ the integral of the (minus) expected price gap until the first time the firms adjusts its price, and starting the economy with a distribution of price gaps with density f we have

$$
\mathcal{M}(\delta) \;=\; \int_{-\bar{p}}^{\bar{p}} m(p - \delta) \; g(p) \, dp \tag{67}
$$

and differentiating it:

$$
\mathcal{M}'(0) \;=\; -\int_{-\bar{p}}^{\bar{p}} m'(p) \; g(p) \, dp \tag{68}
$$

To obtain the solution for m we consider two functions in each segments which solves:

$$0 = -p + \frac{\sigma^2}{2} m_0''(p) \text{ for } 0 \le p \le \underline{p} \tag{69}$$

$$\lambda\, m_1(p) = -p + \frac{\sigma^2}{2} m_1''(p) \text{ for } \underline{p} \le p \le \bar{p} \tag{70}$$

The boundary conditions are that these functions meet in a continuously differentiable manner in the lower boundary, i.e. $m_0(\underline{p}) = m_1(\underline{p})$, $m_0'(\underline{p}) = m_1'(\underline{p})$, and that a price change occurs at the upper boundary, i.e. $m_1(\bar{p}) = 0$. The solution, with three constant of integration is:

$$m_0(p) = A_1\, p + \frac{p^3}{3\,\sigma^2} \tag{71}$$

$$m_1(p) = -\frac{p}{\lambda} + A_2\, e^{p\varphi} + A_3\, e^{-p\varphi} \tag{72}$$

Thus, given $\underline{p}, \bar{p}$ boils down to solving three linear equations in three unknowns, as detailed in Appendix U.6.

Hence, given any pair $(\underline{p}, \bar{p})$ we can find the solution for the density g, the solution to the function m and compute:

$$\mathcal{M}'(0) = -2 \int_0^{\bar{p}} m'(p)\; g(p)\, dp$$

Likewise, given any pair $(\underline{p}, \bar{p})$, we can find the solution for g, $N(\Delta p)$ and compute $Kur(\Delta p)$ as in equation (66). In Appendix U.7 we collect the solutions as function of the thresholds $(\underline{p}, \bar{p})$ and constants $(A_1, A_2, A_3, J, C_3, C_4)$. From this one can easily compute both expressions and check the equality in

$$\mathcal{M}'(0) = \frac{Kur(\Delta p)}{6\, N(\Delta p)} \;.$$

U.1 Solution of ode for value function in inaction

$$v_0(p) = \frac{B\, p^2}{r} + \frac{B\, \sigma^2}{r^2} + K_0 \left(e^{p\sqrt{\frac{2r}{\sigma^2}}} + e^{-p\sqrt{\frac{2r}{\sigma^2}}} \right)$$

$$v_1(p) = \frac{B\, p^2 + \lambda\,(v_0(0) + b\psi)}{\lambda + r} + \frac{B\, \sigma^2}{(\lambda + r)^2} + K_1 \left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} \right)$$

U.2 Solution for value function

Note that smooth pasting $v_1'(\bar{p}) = 0$ gives

$$0 = \frac{2B\bar{p}}{\lambda + r} + K_1\sqrt{\frac{2(\lambda + r)}{\sigma^2}}\left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

or K_1 as function of $\bar{p}$

$$K_1 = \frac{2B\bar{p}}{\lambda + r}\left[\sqrt{\frac{2(\lambda + r)}{\sigma^2}}\left(e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)\right]^{-1} \tag{73}$$

Using $v_0(0) = \frac{B\sigma^2}{r^2} + 2K_0$ and value matching $v_0(0) + \psi = v_1(\bar{p})$ gives

$$\frac{r}{\lambda + r}v_0(0) + \psi = \frac{\lambda b\psi + B\bar{p}^2}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + K_1\left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

or K_0 as function of $\bar{p}$

$$2rK_0 = B\bar{p}^2 - (\lambda(1-b) + r)\psi - \frac{\lambda B\sigma^2}{r(\lambda + r)} + (\lambda + r)K_1\left(e^{\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\bar{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right) \tag{74}$$

Value matching at $\underline{p}$ gives

$$\frac{B\underline{p}^2}{r} + \frac{B\sigma^2}{r^2} + K_0\left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}}\right) = \frac{B\underline{p}^2 + \lambda\left(v_0(0) + b\psi\right)}{\lambda + r} + \frac{B\sigma^2}{(\lambda + r)^2} + K_1\left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

or an equation implicitly defining $\underline{p}$ in terms of $\bar{p}$

$$\frac{B\underline{p}^2\lambda}{r(r+\lambda)} + \frac{B\sigma^2\lambda}{(\lambda + r)^2 r} + K_0\left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}} - \frac{2\lambda}{\lambda + r}\right) = \frac{\lambda b\psi}{\lambda + r} + K_1\left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

Given these 3 equations implicitly defining $K_0, K_1, \underline{p}$ as function of $\bar{p}$, the smooth pasting at $\underline{p}$ gives one equation in one unknown to solve for $\bar{p}$, namely

$$\left(\frac{2B}{r} - \frac{2B}{r+\lambda}\right)\underline{p} + \sqrt{\frac{2r}{\sigma^2}}K_0\left(e^{\underline{p}\sqrt{\frac{2r}{\sigma^2}}} - e^{-\underline{p}\sqrt{\frac{2r}{\sigma^2}}}\right) = \sqrt{\frac{2(\lambda + r)}{\sigma^2}}K_1\left(e^{\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} - e^{-\underline{p}\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right)$$

U.3 Value function approximation

Recall

$$\begin{aligned} v_0(p) &= \frac{Bp^2}{r} + \frac{B\sigma^2}{r^2} + K_0\left(e^{p\sqrt{\frac{2r}{\sigma^2}}} + e^{-p\sqrt{\frac{2r}{\sigma^2}}}\right) \\ v_1(p) &= \frac{Bp^2 + \lambda\left(v_0(0) + b\psi\right)}{\lambda + r} + \frac{B\sigma^2}{(\lambda+r)^2} + K_1\left(e^{p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}} + e^{-p\sqrt{\frac{2(\lambda+r)}{\sigma^2}}}\right) \end{aligned}$$

We approximate the value functions $v_0(p), v_1(p)$ using a fourth order expansion around $p = 0$. We get

$$\begin{aligned} v_0(p) &= \frac{B\sigma^2}{r^2} + 2K_0 + \left(\frac{B}{r} + K_0\varphi_0^2\right) p^2 + \frac{K_0}{12}\varphi_0^4\, p^4 \\ v_1(p) &= \frac{\lambda(v_0(0) + b\psi)}{\lambda + r} + \frac{B\sigma^2}{(\lambda+r)^2} + 2K_1 + \left(\frac{B}{\lambda + r} + K_1\varphi_1^2\right) p^2 + \frac{K_1}{12}\varphi_1^4\, p^4 \\ &\text{where } \varphi_0 \equiv \sqrt{\frac{2r}{\sigma^2}} \quad \text{and} \quad \varphi_1 \equiv \sqrt{\frac{2(\lambda+r)}{\sigma^2}} \end{aligned}$$

The smooth pasting at $\underline{p}$, namely $v_0'(\underline{p}) - v_1'(\underline{p}) = 0$, gives

$$p\left[\left(\frac{B}{r} + K_0\varphi_0^2\right) - \left(\frac{B}{\lambda + r} + K_1\varphi_1^2\right) + \left(K_0\varphi_0^4 - K_1\varphi_1^4\right)\frac{p^2}{6}\right] = 0$$

which gives

$$\underline{p} = \pm\sqrt{\frac{\left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right) - \left(\frac{B}{r} + K_0\varphi_0^2\right)}{\left(K_0\varphi_0^4 - K_1\varphi_1^4\right)/6}}$$

Similarly smooth pasting at $\bar{p}$ gives

$$\bar{p} = \pm\sqrt{\frac{\left(\frac{B}{\lambda+r} + K_1\varphi_1^2\right)}{-K_1\varphi_1^4/6}}$$

U.4 Boundary conditions for T_i

We have the following three linear equations for T_i:

$$
\begin{aligned}
-\frac{1}{\lambda} &= Ke^{\varphi\bar{p}} + Le^{-\varphi\bar{p}} \\
-\frac{2\,\underline{p}}{\sigma^2} &= \varphi\left(Ke^{\varphi\underline{p}} - Le^{-\varphi\underline{p}}\right) \\
J &= \frac{\left(\underline{p}\right)^2}{\sigma^2} + \frac{1}{\lambda} + Ke^{\varphi\underline{p}} + Le^{-\varphi\underline{p}}
\end{aligned}
$$

U.5 Density function

The 4 unknowns of the density function, using $g_1(\bar{p}) = 0$ and $g_0'(\underline{p}) = g_1'(\underline{p})$, give

$$C_3 = -C_4 e^{-2\varphi\bar{p}} \quad \text{and} \quad C_2 = -C_4\varphi\left(e^{-2\varphi\bar{p}+\varphi\underline{p}} + e^{-\varphi\underline{p}}\right)$$

Next, using $g_0(\underline{p}) = g_1(\underline{p})$ gives

$$C_1 = -C_2\underline{p} - C_4\left(e^{-2\varphi\bar{p}+\varphi\underline{p}} - e^{-\varphi\underline{p}}\right) = C_4\left[e^{-2\varphi\bar{p}+\varphi\underline{p}}\left(\varphi\underline{p} - 1\right) + e^{-\varphi\underline{p}}\left(\varphi\underline{p} + 1\right)\right]$$

Finally we solve for C_4 by imposing $1/2 = \int_0^{\underline{p}} g_0(p)\,\mathrm{d}p + \int_{\underline{p}}^{\bar{p}} g_1(p)\,\mathrm{d}p$ i.e.

$$\frac{1}{2} = C_1\underline{p} + \frac{1}{2}C_2\underline{p}^2 + \frac{1}{\varphi}\left[C_3\left(e^{\varphi\bar{p}} - e^{\varphi\underline{p}}\right) - C_4\left(e^{-\varphi\bar{p}} - e^{-\varphi\underline{p}}\right)\right]$$

or, substituting the expressions,

$$
\begin{aligned}
\frac{1}{2C_4} &= \left[e^{-2\varphi\bar{p}+\varphi\underline{p}}\left(\varphi\underline{p} - 1\right) + e^{-\varphi\underline{p}}\left(\varphi\underline{p} + 1\right)\right]\underline{p} - \frac{1}{2}\varphi\left(e^{-2\varphi\bar{p}+\varphi\underline{p}} + e^{-\varphi\underline{p}}\right)\underline{p}^2 \\
&\quad -\frac{1}{\varphi}\left[e^{-2\varphi\bar{p}}\left(e^{\varphi\bar{p}} - e^{\varphi\underline{p}}\right) + e^{-\varphi\bar{p}} - e^{-\varphi\underline{p}}\right]
\end{aligned}
$$

U.6 Equation for the solution of m

The boundary conditions are: $m_1(\bar{p}) = 0$, $m_1(\underline{p}) = m_0(\underline{p})$ and $m_1'(\underline{p}) = m_0'(\underline{p})$. They give a linear system of equations on A_1, A_2, A_3 :

$$0 = -\frac{\bar{p}}{\lambda} + A_2\,e^{\bar{p}\varphi} + A_3\,e^{-\bar{p}\varphi} \tag{75}$$

$$A_1 + \frac{(\underline{p})^2}{\sigma^2} = -\frac{1}{\lambda} + \varphi\,A_2\,e^{\underline{p}\varphi} - \varphi\,A_3\,e^{-\underline{p}\varphi} \tag{76}$$

$$A_1\,\underline{p} + \frac{(\underline{p})^3}{3\,\sigma^2} = -\frac{\underline{p}}{\lambda} + A_2\,e^{\underline{p}\varphi} + A_3\,e^{-\underline{p}\varphi} \tag{77}$$

U.7 Algebraic details for main proposition

For the area under the IRF of output we get:

$$\begin{aligned}\mathcal{M}'(0) = & -\; 2\int_0^{\underline{p}} \left[A_1 + A_3\frac{p^2}{\sigma^2}\right][C_1 + C_2 p]\,\mathrm{d}p \\ & -\; 2\int_{\underline{p}}^{\bar{p}} \left[-\frac{1}{\lambda} + \varphi\, A_2\, e^{p\varphi} + \varphi\, A_3\, e^{-p\varphi}\right]\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p\end{aligned}$$

For the kurtosis of steady state price changes we get:

$$\begin{aligned}\frac{Kur\,(\Delta p)}{6\,N\,(\Delta p)} &= N\,(\Delta p)\,\frac{\mathbb{E}\,(\Delta p^4)}{6\,\sigma^4} \\ &= \frac{\lambda\, J\, 2\int_{\underline{p}}^{\bar{p}} p^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p + \left(1 - \frac{\lambda\, 2\int_{\underline{p}}^{\bar{p}}\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p}{N_a}\right)\bar{p}^4}{6\,J\,\sigma^4} \\ &= \frac{\lambda\, 2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} p^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p + \frac{1}{6\,J\,\sigma^4} - \frac{\lambda\, 2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} \bar{p}^4\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p \\ &= \frac{\lambda\, 2}{6\,\sigma^4}\int_{\underline{p}}^{\bar{p}} \left(p^4 - \bar{p}^4\right)\,\left[C_3 e^{\varphi p} + C_4 e^{-\varphi p}\right]\mathrm{d}p + \frac{1}{6\,J\,\sigma^4}\end{aligned}$$